H2 MATHEMATICS SUMMARY NOTES

First Edition

Σ

$\frac{dy}{dx}$

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TIMGANMATH
WHERE PASSIONATE TEACHING INSPIRES

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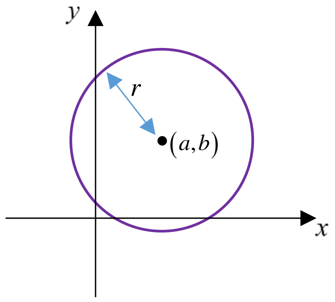
PURE MATHEMATICS

STATISTICS

CONICS GRAPHS

CIRCLE

$$(x-a)^2+(y-b)^2=r^2 \text{ or}$$

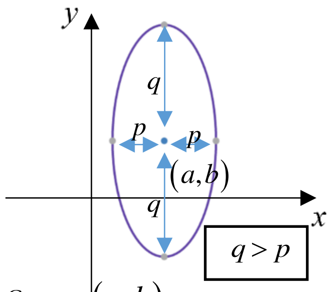


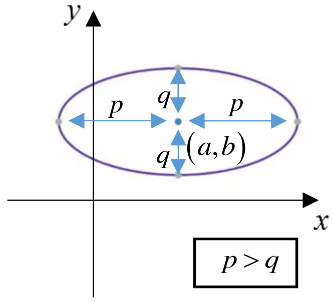
Centre: (a,b)

Radius: r

ELLIPSE

$$\frac{(x-a)^2}{p^2}+\frac{(y-b)^2}{q^2}=1$$

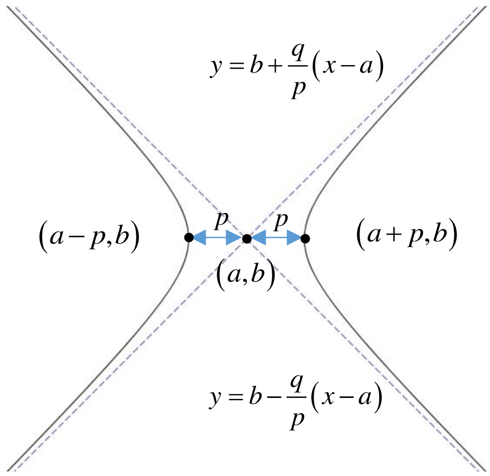


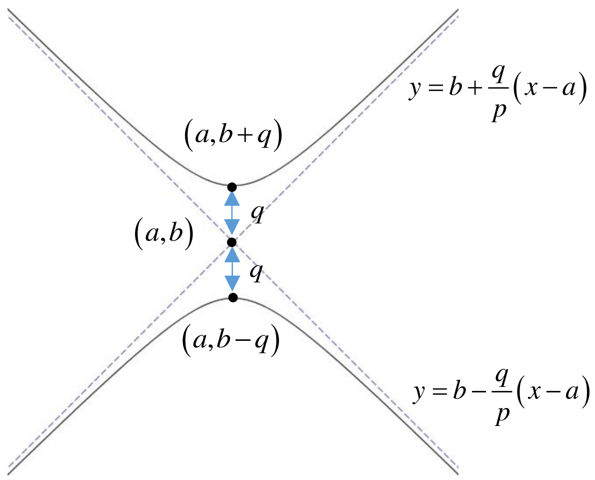


Centre: (a,b)

HYPERBOLA

$$\frac{(x-a)^2}{p^2}-\frac{(y-b)^2}{q^2}=1 \qquad \frac{(y-b)^2}{q^2}-\frac{(x-a)^2}{p^2}=1$$





Finding the oblique asymptotes: As $x \to \infty$, $\frac{(x-a)^2}{p^2} \approx \frac{(y-b)^2}{q^2}$. [Equate to obtain the equations of asymptotes.]

Gradients of Oblique asymptotes: $y = \pm\frac{q}{p}$

TRANSFORMATION OF GRAPHS

$y = f(x)$	Vertical Transformation	Horizontal Transformation
Translation	a units in the **positive** y direction: Replace y with $y-a$ $y-a=f(x) \Leftrightarrow y=f(x)+a$	a units in the **positive** x direction: Replace x with $x-a$ $y=f(x-a)$
	a units in the **negative** y direction: Replace y with $y+a$ $y+a=f(x) \Leftrightarrow y=f(x)-a$	a units in the **negative** y direction: Replace x with $x+a$ $y=f(x+a)$
Scaling	Scaling by a factor of a parallel to the *y*-axis: Replace y with $\frac{y}{a}$ $\frac{y}{a}=f(x) \Leftrightarrow y=af(x)$	Scaling by a factor of a parallel to the *y*-axis: Replace x with $\frac{x}{a}$ $y=f\left(\frac{x}{a}\right)$
Reflection	Reflection about the x -axis: Replace y with $-y$ $-y=f(x) \Leftrightarrow y=-f(x)$	Reflection about the y -axis: Replace x with $-x$ $y=f(-x)$
Modulus	$y=\lvert f(x)\rvert$ Reflect the negative *y* portion to positive about the *x* axis. Omit the negative y portion (where $y<0$).	$y=f(\lvert x\rvert)$ Omit the negative *x* portion. (where $x<0$) Retain the positive *x* portion of the graph and reflect it about the *y* axis.

COMPOSITE TRANSFORMATIONS

$y=af(bx+c)+d$	$y=f(x)\xrightarrow{\text{I}}y=f(x+c)\xrightarrow{\text{II}}y=f(bx+c)\xrightarrow{\text{III}}y=af(bx+c)\xrightarrow{\text{IV}}y=af(bx+c)+d$ Sequence of Transformations: I: Translation of c units in the negative x direction. (Replace x with $x+c$) II: Scaling with a factor of $\frac{1}{b}$ parallel to the x axis. (Replace x with bx) III: Scaling with a factor of a parallel to the y axis. (Replace y with $\frac{y}{a}$) IV: Translation of d units in the positive y direction. (Replace y with $y-d$)
$y=f(a-\lvert x\rvert)$	$y=f(x)\xrightarrow{\text{I}}y=f(x+a)\xrightarrow{\text{II}}y=f(-x+a)\xrightarrow{\text{III}}y=f(-\lvert x\rvert+a)$ Sequence of Transformations: I: Translation of a units in the negative x direction. (Replace x with $x+a$) II: Reflection about the y axis. (Replace x with $-x$) III: Perform $y=f(\lvert x\rvert)$. (Replace x with $\lvert x\rvert$)

RECIPROCAL GRAPH $y=\frac{1}{\mathrm{f}(x)}$

Original Graph, $y=\mathrm{f}(x)$ $\Rightarrow$ Reciprocal Graph, $y=\frac{1}{\mathrm{f}(x)}$	
MARKERS	Point (x,y) $\Rightarrow$ Point $\left(x,\frac{1}{y}\right)$
	x - intercept $\Rightarrow$ Vertical Asymptote Vertical Asymptote $\Rightarrow$ x - intercept
	Maximum point $\Rightarrow$ Minimum point Minimum point $\Rightarrow$ Maximum point
	Horizontal Asymptote, $y=a$ $\Rightarrow$ Horizontal Asymptote, $y=\frac{1}{a}$ **(Except Horizontal Asymptote, $y=0$ or x -axis)**
SKETCH	$\mathrm{f}(x)>0 \Rightarrow \frac{1}{\mathrm{f}(x)}>0$ $\mathrm{f}(x)<0 \Rightarrow \frac{1}{\mathrm{f}(x)}<0$ If original graph is below/above the x axis, then the reciprocal graph remains below/above x axis.
	Curve approaches Horizontal Asymptote from above $\Rightarrow$ approaches Horizontal Asymptote from below and **vice versa. (Except Horizontal Asymptote, $y=0$ or x -axis)**
	$y\to 0 \Rightarrow y\to\infty$ $y\to\infty \Rightarrow y\to 0$
	Increasing $\Rightarrow$ Decreasing Decreasing $\Rightarrow$ Increasing

GRAPH OF THE DERIVATIVE FUNCTION $y=\mathrm{f}'(x)$ (GRADIENT GRAPH)

Original Graph, $y=\mathrm{f}(x)$ $\Rightarrow$ Gradient Graph, $y=\mathrm{f}'(x)$	
MARKERS	Vertical Asymptote remains the same.
	Horizontal Asymptote, $y=a$ $\Rightarrow$ Horizontal Asymptote, $y=0$ (x -axis)
	Oblique Asymptote, $y=mx+c$ $\Rightarrow$ Horizontal Asymptote, $y=m$
	Stationary point (a,b) $\Rightarrow$ x -intercept $(x=a)$
	Point of inflexion (Increasing/ Decreasing function) $\Rightarrow$ Turning Point (Max/ Min)
SKETCH	Compartmentalize graph with regions of positive or negative gradients. If gradient of $y=\mathrm{f}(x)$ is positive, draw above the x -axis. If gradient of $y=\mathrm{f}(x)$ is negative, draw below the x -axis.
	If $y=\mathrm{f}(x)$ approaches horizontal asymptote, then $\mathrm{f}'(x)$ approaches $y=0$. (x -axis) If $y=\mathrm{f}(x)$ approaches oblique asymptote, $y=mx+c$, then $\mathrm{f}'(x)$ approaches $y=m$. If gradient of $y=\mathrm{f}(x)$ approaches ∞, then $\mathrm{f}'(x)$ approaches vertical asymptote.

INEQUALITIES

BASIC RESULTS

1. If $a>b$ and $b>c \Rightarrow a>c$ — E.g. $12>5$ and $5>-1 \Rightarrow 12>-1$
2. If $a>b \Rightarrow a\pm c>b\pm c$ — E.g. $-5>7 \Rightarrow -5\pm10>7\pm10$
3. If $a>b$ and $c>0 \Rightarrow ac<bc$; $\frac{a}{c}<\frac{b}{c}$ — E.g. $9<21 \Rightarrow \frac{9}{2}<\frac{21}{2}$
4. If $a>b$ and $c<0 \Rightarrow ac<bc$; $\frac{a}{c}>\frac{b}{c}$ — E.g. $9<21 \Rightarrow -\frac{9}{2}>-\frac{21}{2}$
5. If $ab>0 \Rightarrow$ "$a<0$ and $b<0$" or "$a>0$ and $b>0$"
6. If $ab<0 \Rightarrow$ "$a<0$ and $b>0$" or "$a>0$ and $b<0$"

Important notes:

1. Do not multiply inequality equations with variables without knowing whether it's positive or not.
2. Examples of variables which can be multiplied both sides with: $|x|$, e^{-x}, $(3x-1)^2$
3. Know the difference between "**and**" and **"or"**. I.e. intersection and union of sets.
4. Solutions should not be equal to the roots of the denominator.

MODULUS FUNCTION

$\lvert x\rvert<a \Leftrightarrow -a<x<a$, where a is a positive constant	$\lvert x\rvert>a \Leftrightarrow x<-a$ **or** $x>a$, where a is a positive constant	$a<x<b \Leftrightarrow a<x$ **and** $x<b$

Important notes:

1. $x^2=|x|^2$.
2. If both sides of the inequality are positive, then you can **square** both sides. i.e. $|a|<|b| \Leftrightarrow a^2<b^2$
3. Do not **square** both sides if both sides are **not** positive. i.e. $|x|<x+1 \neq x^2<(x+1)^2$. Solve it graphically.

INEQUALITIES INVOLVING SUBSTITUTION

Given that $x<-a$ or $x>b$ where a and b are positive constants.

Substitute x as e^x $\Rightarrow e^x<-a$ (N.A) or $e^x>b$ $\Rightarrow e^x>b$ $\Rightarrow x>\ln b$	Substitute x as $\ln x$ $\Rightarrow \ln x<-a$ or $\ln x>b$ $\Rightarrow 0<x<e^{-a}$ or $x>e^b$ Note: $x>0$ for $\ln x$	Substitute x as $\lvert x\rvert$ $\Rightarrow \lvert x\rvert<-a$ (N.A) or $\lvert x\rvert>b$ $\Rightarrow \lvert x\rvert>b$ $\Rightarrow x>b$ or $x<-b$

Given that $-a<x<b$ where a and b are positive constants.

Substitute x as e^x $\Rightarrow -a<e^x<b$ $\Rightarrow 0<e^x<b$ $\Rightarrow x<\ln b$	Substitute x as $\ln x$ $\Rightarrow -a<\ln x<b$ $\Rightarrow e^{-a}<x<e^b$	Substitute x as $\lvert x\rvert$ $\Rightarrow -a<\lvert x\rvert<b$ $\Rightarrow 0\le\lvert x\rvert<b$ $\Rightarrow 0\le\lvert x\rvert$ and $\lvert x\rvert<b$ $\Rightarrow -b<x<b$

TESTIMONIALS FROM STUDENTS

Mr. Gan is a really genuine and enthusiastic teacher. He puts in the extra hours to ensure our lessons are well-prepared and that his resources are conveniently accessible which allows us to learn much more efficient.

Emma Tang, CJC

Mr. Gan also provides numerous ways to approach a question, allowing me to choose the one that I'm most comfortable with during application.

Mai Goh, AJC

I always look forward to math tuition with Mr Gan because he is a super entertaining teacher. He creates a comfortable and fun environment for us to learn math and even make friends from other schools.

Sarita Zhang, HCI

He teaches with enthusiasm and passion and has a very efficient way of teaching using his tablet which is effective yet simple.

Ng Shuherng, NYJC

He simplified tedious formulas and thought of alternate methods to help us. Now I can do math in a more confident way and can even help others with their doubts.

Sangari, AJC

Mr Gan's lessons are well structured and organised. He covers a wide range of questions from each topic. I feel that I am able to grasp topics better as Mr Gan explains concepts clearly.

Abirami, RJC