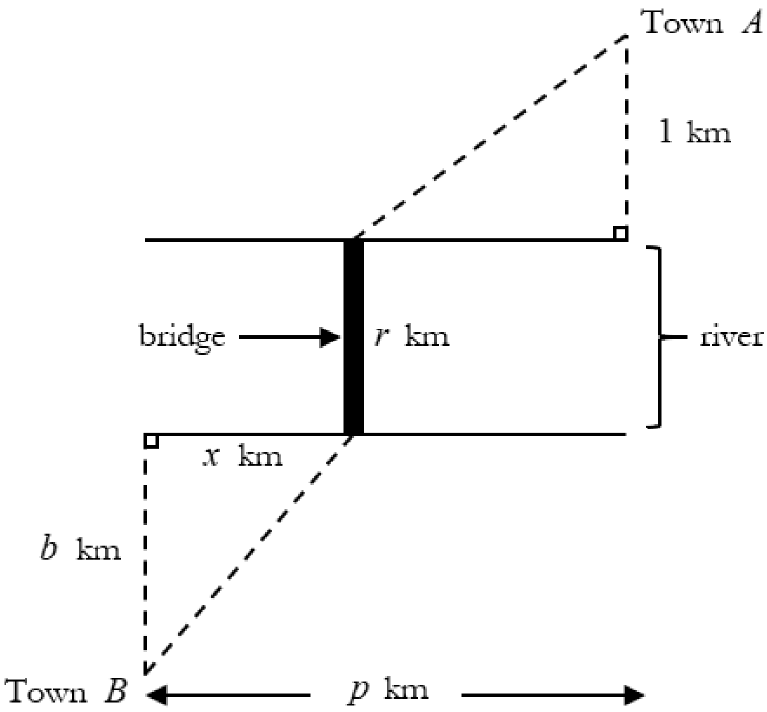
Applications of Differentiation

2020 NYJC CT2 P2 Q3

The diagram below shows a city map of two towns, A and B separated by a river. A bridge is to be built between the two towns, which are on opposite sides of a straight river of uniform width r km, and the two towns are p km apart measured along the riverbank. Town A is 1 km from the riverbank, and Town B is b km away from riverbank.

A bridge is to be built perpendicular to the riverbank at a distance of x km from Town B, measured along the riverbank, allowing traffic to flow between the two towns.

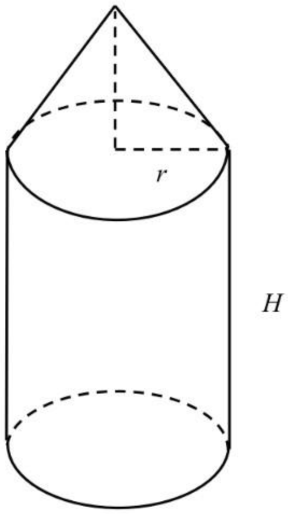


Find the distance x, in terms of b and p, such that the distance of travel between Town A and Town B can be minimised if $b > 1$. (It is not necessary to verify that the distance is minimum.) [9]

Answer: $x = \dfrac{bp}{1+b}$

2019 DHS PROMO Q12

[A circular cone with base radius r, vertical height h and slant height l, has curved surfaced area πrl and volume $\frac{1}{3}\pi rh^2$.]



A capsule made of metal sheet of fixed volume p cm^3 is made up of three parts.

- The top is modelled by the curved surface of a circular cone of radius r cm. The ratio of its height to its base radius is $4:3$.
- The body is modelled by the curved surface of a cylinder of radius r cm and height H cm.
- The base is modelled by a circular disc of radius r cm.

The cost of making the body of the capsule is \$$k$ per cm^2, while that of the top and the base of the capsule is \$$2k$ per cm^2, where k is a constant. The total cost of making the capsule is \$ C.

Assume the metal sheet is made of negligible thickness.

(i) Show that $H = \dfrac{p}{\pi r^2} - \dfrac{4r}{9}$. [2]

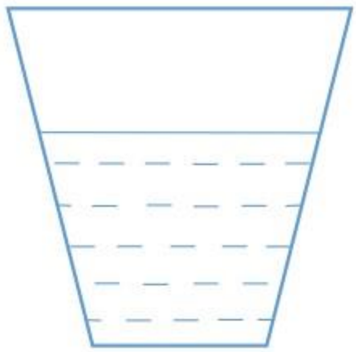
(ii) Express C in the form $\dfrac{A}{r} + Br^2$, where A and B are expressions in terms of k and p. Use differentiation to show that C has a minimum as r varies. [8]

(iii) Hence determine the ratio of H to r when C is a minimum. [2]

Answers: (ii) $C = \dfrac{2kp}{r} + \dfrac{40}{9}\pi r^2 k$ (iii) $4:1$

2007 AJC P1 Q13 (B)

Water is poured at a constant rate of $20\,\text{cm}^3$ per second into a cup which is shaped like a truncated cone as shown in the figure. The upper and lower radii of the cup are $4\,\text{cm}$ and $2\,\text{cm}$ respectively. The height of the cup is $6\,\text{cm}$.



(i) Show that the volume of water inside the cup, V is related the height of the water level, h through the equation

$$V = \frac{\pi}{27}(h=6)^3 - 8\pi$$

[3]

(ii) How fast will the water level be rising when h is $3\,\text{cm}$?

Express your answer in exact form. [3]

Answer: (ii) $\dfrac{20}{9\pi}$ cm/s

2018 MJC P1 Q9 (II) MODIFIED

A manufacturer produces closed hollow cans. The top part is a hemisphere made of tin and the bottom part is a cylinder made of aluminium of cross-sectional radius r cm and h cm. There is no material between the cylinder and the hemisphere so that any fluid can move freely within the container. At the beginning of an experiment, a similar-shaped can of dimensions $r = 4$ and $h = 10$, is filled to its capacity with water. Due to a hole at its base, water is leaking at a constant rate of $2\,\text{cm}^3\text{s}^{-1}$ when the can is standing upright. Find the exact rate at which the height of the water is decreasing 80 seconds after the start of the experiment. [5]

Answer: $\dfrac{1}{8\pi}$ cm/s

MAXIMUM AREA OF ISOSCELES TRIANGLE

An isosceles triangle is inscribed in a circle of radius r. Find the maximum area of this triangle in terms of r.

Answer: $\dfrac{3}{4}\sqrt{3}r^2$ units^2