BINOMIAL DISTRIBUTION

2015 DHS P2 Q7 [MODIFIED]

Data is transmitted in bytes, where each byte consists of 8 bits. The probability of a bit being corrupted during its transmission is 0.03. A byte is considered 'corrupted' if it contains at least 2 corrupted bits.

Assume that all bits are not corrupted prior to their transmission.

(i) Find the probability that a randomly chosen byte is corrupted during its transmission.

(ii) Given that a randomly chosen byte is not corrupted during its transmission, find the probability that it contains no corrupted bits.

(iii) Find the probability that between 5 and 10 bytes are corrupted during the transmission of 100 bytes.

In another transmission system, the probability of having at least 3 corrupted bytes during the transmission of 20 bytes is 0.85. Find the probability that a bit is corrupted during its transmission.

Answer: (i) 0.0233 (ii) 0.802 (iii) $p_2 = 0.111$

2017 VJC J2 MYE P1 Q11

A trial for a new treatment for a particular disease is to be carried out. A sample of n patients suffering from this disease is taken and these patients are invited to take part in the trial. The number of patients in the sample who accept the invitation is denoted by X.

(i) State, in context, two assumptions needed for X to be well modelled by a binomial distribution. [2]

Assume now that X has the distribution $\mathrm{B}(10, 0.25)$.

(ii) Find the most probable number of patients who accept the invitation. [2]

(iii) Find the probability that exactly 6 patients accept the invitation given that at least 1 patient accepts the invitation. [2]

Answers: (ii) 2 (iii) 0.0172 (3 s.f.)

N1982 P1 Q13

The random variable X is the number of successes in n independent trials of an experiment in which the probability of a success in any one trial is p.

Show that $\dfrac{\mathrm{P}(X=k+1)}{\mathrm{P}(X=k)} = \dfrac{(n-k)p}{(k+1)(1-p)}, k = 0, 1, 2, \ldots, (n-1)$

Find the most probable number of success when $n = 10$ and $p = \dfrac{1}{4}$.

Answer: 2

2018 ACJC MYE Q8

The random variable X is the number of successes in n independent trials of an experiment in which the probability of a success at any trial is p.

Denoting $\mathrm{P}(X=k)$ by p_k, it is given that $\dfrac{p_k}{p_{k-1}}=\dfrac{(n-k+1)p}{k(1-p)}$, $k=1, 2, \ldots, n$.

A distribution is said to be bimodal if it has two modes.

Find the least value of n, and the corresponding modes of X, given that X is bimodal and that $p=\dfrac{18}{25}$.

Answer: 17,18

2019 SAJC P2 Q10

A factory manufactures a large number of erasers in a variety of colours. Each box of erasers contains 36 randomly chosen erasers. On average, 20% of erasers in the box are blue.

(i) State, in context, two assumptions needed for the number of blue erasers in a box to be well modelled by a binomial distribution. [2]

(ii) Find the probability that a randomly chosen box of erasers contain at most six blue erasers. [1]

200 randomly chosen boxes are packed into a carton. A carton is considered acceptable if at least 40% of the boxes contain at most six blue erasers each.

(iii) Find the probability that a randomly chosen carton is acceptable. [3]

The cartons are exported by sea. Over a one-year period, there are 30 shipments of 150 cartons each.

(iv) Using a suitable approximation, find the probability that the mean number of acceptable cartons per shipment for the year is less than 80. [3]

The owner decided to change the proportion of blue erasers to p. A box of erasers is chosen.

(v) Write down in terms of p, the probability that the box contains exactly one blue eraser. [1]

(vi) The probability that a box contains exactly one blue eraser is twice the probability that the box contains exactly two blue erasers. Write an equation in terms of p, and hence find the value of p. [2]

Answers: (i) Every eraser is equally likely to be blue; The colour of a randomly selected eraser is independent of the colour of other erasers. (ii) 0.401 (iii) 0.535 (iv) 37.319 (v) $36p(1-p)^{35}$ (vi) $\dfrac{1}{36}$