COMPLEX NUMBERS

2017 NYJC P1 Q3

Do not use a calculator in answering this question.

(i) Explain why the equation $z^3+az^2+az+7=0$ cannot have more than two non-real roots, where a is a real constant. [1]

(ii) Given that $z=-7$ is a root of the equation in (i), find the other roots, leaving your answers in the form $re^{i\theta}$, where $r>0$ and $-\pi<\theta\leq\pi$. [4]

(iii) Hence, solve the equation $iz^3+8z^2-8iz-7=0$, leaving your answers in the form $re^{i\theta}$, where $r>0$ and $-\pi<\theta\leq\pi$. [2]

Answers: (ii) $z=7e^{i\pi}, e^{\frac{i2\pi}{3}}, e^{-\frac{i2\pi}{3}}$ (iii) $z=7e^{\frac{i\pi}{2}}, e^{\frac{i\pi}{6}}, e^{\frac{i5\pi}{6}}$

2009 NJC P1 Q9 (B) MODIFIED

Write down the real root of

$$w^3+8=0,$$

Where w is a complex number.

Hence, find the other roots of $w^3+8=0$ in exact form.

Deduce the roots of $w^4+w^3+8w+8=0$.

Determine the roots of z such that $z^4-\mathbf{i}z^3+8\mathbf{i}z+8=0$.

Answers: $w=-2$; $1+\sqrt{3}\mathbf{i}$, $1-\sqrt{3}\mathbf{i}$; -1, -2, $1+\sqrt{3}\mathbf{i}$, $1-\sqrt{3}\mathbf{i}$; $z=\mathbf{i}$, $2\mathbf{i}$, $-\mathbf{i}+\sqrt{3}$, $-\mathbf{i}-\sqrt{3}$

2014 JJC P1 Q10 (A)

Given that the complex number $z=(1+i)t+\frac{1-i}{t}$ is represented by the point P on Argand diagram where t is a non-zero real constant. Find the Cartesian equation of the locus of the point P. [3]

Answer: $x^2-y^2=4$

2020 CJC P1 Q2

The complex number z has modulus 2 and argument $\frac{\pi}{8}$. It is also given that $w=1+\mathrm{i}$.

(i) Given that n is an integer, find $\frac{z^n}{w^*}$ in terms of n, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$. [3]

(ii) Hence, find the smallest two positive integers n such that $\frac{z^n}{w^*}$ is real and negative. [3]

Ans: (i) $2^{n-\frac{1}{2}}\mathrm{e}^{i\left(\frac{n\pi}{8}+\frac{\pi}{4}\right)}$ (ii) Smallest $n=6,22$

2015 RI P1 Q6 (B) MODIFIED

Prove that $\dfrac{1+\sin\frac{3\pi}{8}+\mathrm{i}\cos\frac{3\pi}{8}}{1+\sin\frac{3\pi}{8}-\mathrm{i}\cos\frac{3\pi}{8}}=\cos\frac{\pi}{8}+\mathrm{i}\sin\frac{\pi}{8}$.