GRAPHING TECHNIQUES

QUESTION 1

The curve C has equation $y=\dfrac{x^2+ax+b}{x+c}$, where a, b and c are constants. The line $x=3$ is an asymptote to C and the range of values that y can take is given by $y\le -2$ or $y\ge 4$. Find the values of a, b and c. [4]

Answer: $a=-5, b=\dfrac{33}{4}, c=-3$

ACJC TUTORIAL 2 Q8

It is given that $y=\dfrac{2x^2+3}{x-2}, x\in\mathbb{R}, x\ne 2$.

(i) Sketch the graph of $y=\dfrac{2x^2+3}{x-2}$ and label clearly the equations of the asymptotes and intercepts if any.

(ii) The graph of $y=\dfrac{2x^2+3}{x-2}$ cannot lie between values p and m, where $p>m$. State the value of p and m.

(iii) By means of a graphical argument, state the maximum number of real roots the equation $kx(x-2)-2x^2=3$ have.

Answers: (ii) $p=8+2\sqrt{22}$; $m=8-2\sqrt{22}$ (iii) Maximum number of real roots: 2

2013 YJC P1 Q12

The curve C has equation $y=\dfrac{x^2}{x-2}$.

(i) Find the equation(s) of the asymptote(s) of C. [1]

(ii) Sketch the curve C, labelling the equation(s) of its asymptote(s) and coordinates of any axial intercepts and turning points. [2]

(iii) Hence find the range of values of k for which the equation $x^2=k\left(x^2-4\right)$ has no real roots.

Answers: (i) $x=2$, $y=x+2$ (iii) $0<k\le 1$

2013 MJC Promo Q6

The curve C has equation $y=\sqrt{5x^2+4}$.

(i) Sketch curve C, indicating clearly the axial intercepts, the equations of the asymptotes and the coordinates of the stationary points.

(ii) Hence, by inserting a suitable graph, determine the range of values of h, where h is a positive constant, such that the equation $\sqrt{5x^2+4}=h\sqrt{(1-x^2)}$ has no real roots.

Answer: (ii) $0<h<2$

2009 SAJC P1 Q7 [Modified]

The curve C has equation $y=\dfrac{ax^2+2}{x-1}$ where $x\neq 1$ and a is a non-zero constant.

(i) Show that if the curve C has no stationary points, then $-2<a<0$.

(ii) Sketch the curve for $a=1$, showing clearly the asymptotes and coordinates of any intersections with the coordinate axes.

(iii) Verify that $y=k(x-1)+2$ passes through $(1,2)$ for all real values of k.

(iv) By considering the equation of an appropriate line drawn on the same diagram with the curve C, find the range of values of k for which the equation $x^2+2=k(x-1)^2+2(x-1)$ has no real roots.

Answer: $k\leq 1$