MACLAURIN AND POWER SERIES

2009 HCI PROMO P1 Q3 [MODIFIED]

Expand $\dfrac{x^2+2x}{2x^2+1}$ in ascending powers of x up to and including the term in x^5. State the range of values of x for which this expansion is valid. [3]

Find, in the simplest form, the coefficient of x^{2017} in this expansion. [2]

Answer: (i) $2x+x^2-4x^3-2x^4+8x^5+\ldots$ (ii) 2^{1009}

2020 MI P1 Q6

It is given that $y=\sqrt{\mathrm{e}^x\cos x}$.

(i) Show that $2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=y^2-\mathrm{e}^x\sin x$. [2]

(ii) By further differentiation of the result in part **(i)**, find the Maclaurin series for y, up to and including the term in x^2. [4]

(iii) Using the standard series from the List of Formulae (MF 26). Expand $\sqrt{\mathrm{e}^x\cos x}$ as far as the term in x^2 and verify that the same result is obtained in part **(ii)**. [3]

Answer: (ii) $y=1+\dfrac{1}{2}x-\dfrac{1}{8}x^2+\ldots$

2013 ACJC P1 Q12

Given that $y=\left(\sin^{-1}x\right)^2$, show that

$$\left(1-x^2\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=4y$$

and $\left(1-x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\dfrac{\mathrm{d}y}{\mathrm{d}x}=2$.

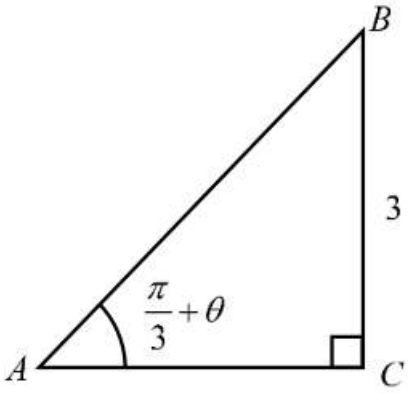
[3]

By further differentiation of these results, find the Maclaurin series of y up to including the term in x^4. [3]

Answer: $x^2+\dfrac{1}{3}x^4+\ldots$

2017 PJC PROMO Q9 (A)

In the triangle ABC as shown below, $BC = 3$, angle $BAC = \frac{\pi}{3} + \theta$ radians and angle $ACB = \frac{\pi}{2}$ radians.



Show that $AC = \dfrac{3\left(1 - \sqrt{3}\tan\theta\right)}{\sqrt{3} + \tan\theta}$. [3]

Given that θ is a sufficiently small angle, deduce that $AC \approx \sqrt{3} + a\theta$, where a is a constant to be determined.

[3]

Answer: $a = -4$

PROBLEMS INVOLVING STANDARD SERIES

Using the standard series in MF26, show $e^{i\theta} = \cos\theta + i\sin\theta$.