VECTORS

2011 MJC P1 Q7

The lines l_1 and l_2 have equations $\mathbf{r}=\overrightarrow{OA}+\lambda\begin{pmatrix}1\\-1\\5\end{pmatrix}$ and $\mathbf{r}=\overrightarrow{OB}+\mu\begin{pmatrix}2\\1\\1\end{pmatrix}$, λ, $\mu\in\mathbb{R}$ respectively, where $\overrightarrow{OA}=\begin{pmatrix}1\\2\\-1\end{pmatrix}$, $\overrightarrow{OB}=\begin{pmatrix}2\\4\\-5\end{pmatrix}$. The lines l_1 and l_2 intersect at point C.

(i) Find the position vector of C. [3]

(ii) Given that $\overrightarrow{AB}$ is perpendicular to l_2, find the equation of the reflection of l_1 in l_2. [4]

Answers: (i) $\overrightarrow{OC}=\begin{pmatrix}0\\3\\-6\end{pmatrix}$ (ii) $l:\mathbf{r}=\begin{pmatrix}3\\6\\-9\end{pmatrix}+\alpha\begin{pmatrix}1\\1\\-1\end{pmatrix};\alpha\in\mathbb{R}$

2012 CJC PROMO Q10

The lines l_1 passes through the points A and B with coordinates $(-1,-2,1)$ and $(0,1,5)$ respectively. The line l_2 has equation $x-1=\dfrac{y-2}{2}=z-3$. l_1 and l_2 intersect at point A. Find

(i) the vector equations of the lines l_1 and l_2. [2]

(ii) the acute angle between the lines l_1 and l_2. [2]

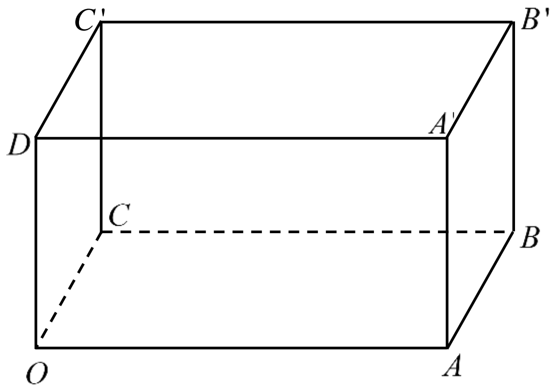
(iii) the position vector of the foot of perpendicular, F, from B to the line l_2. [3]

(iv) the equation of the line l_3 which is the mirror image of l_1 in l_2. [3]

Answers: (i) $l_1:\mathbf{r}=\begin{pmatrix}0\\1\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\3\\4\end{pmatrix};\lambda\in\mathbb{R}$, $l_2:\mathbf{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+\mu\begin{pmatrix}1\\2\\1\end{pmatrix};\mu\in\mathbb{R}$ (ii) $28.3°$ (3 d.p.) (iii) $\overrightarrow{OF}=\frac{1}{6}\begin{pmatrix}5\\10\\17\end{pmatrix}$

(iv) $l_3:\mathbf{r}=\begin{pmatrix}-1\\-2\\1\end{pmatrix}+\alpha\begin{pmatrix}8\\13\\-1\end{pmatrix};\alpha\in\mathbb{R}$

ACJC TUTORIAL 15 Q9



The diagram shows a cuboid $OABCA'B'C'D'$ in which the lengths of OA, OC and OD' are $3a$, $2a$ and a respectively, where $a \neq 0$. The point O is taken as origin, with unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ in the direction of $\overrightarrow{OA}$, $\overrightarrow{OC}$, $\overrightarrow{OD'}$ respectively. The points P on $C'B'$ and Q on OD' are such that $C'P = \frac{1}{3}C'B'$ and $OQ = \frac{1}{3}OD'$ respectively.

(i) Find the position vectors of P and Q.

(ii) Hence find the vector equation of line PQ.

(iii) Determine whether the lines PQ and AC' are skew lines.

(iv) Find the shortest distance from M, the midpoint of AB' to the line PQ.

(v) Use vector product to find the area of triangle PQM.

Answers: (i) $\overrightarrow{OP} = a\mathbf{i} + 2a\mathbf{j} + a\mathbf{k}$, $\overrightarrow{OQ} = \frac{1}{3}a\mathbf{k}$ (ii) $l_{PQ} : \mathbf{r} = \begin{pmatrix} a \\ 2a \\ a \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix}; \lambda \in \mathbb{R}$

(iv) $\frac{5}{14}\sqrt{41}a$ units (v) $\frac{5a^2}{12}\sqrt{41}$ units^2

2020 TJC P2 Q4

(a) Show that the triangle with vertices A, B and C is an isosceles right-angled triangle.

(i) It is given that $\mathbf{a} \times \mathbf{b} = \mathbf{0}$. Find $\mathbf{a} \cdot \mathbf{b}$ in terms of $|\mathbf{b}|$.

It is now given that $\mathbf{a} \times \mathbf{b} \neq \mathbf{0}$. Point P is the foot of the perpendicular from A to the line OB and the point Q is the foot of the perpendicular from B to the line OA. It is given that $AP = BQ$.

(ii) Write down the lengths AP and BQ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

Hence show that $|\mathbf{a}| = |\mathbf{b}|$. [1]

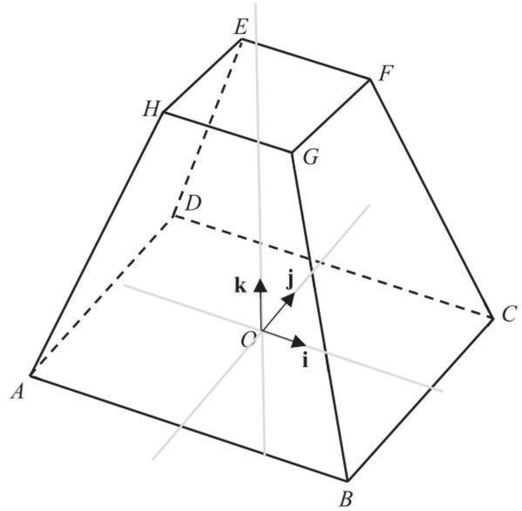
(iii) Given also that the angle between $\mathbf{a}$ and $\mathbf{a} - \mathbf{b}$ is ϕ radians, show that $\mathbf{a} \cdot \mathbf{b} = -|\mathbf{a}|^2 \cos 2\phi$. [2]

(b) The pyramid $ORST$ has a triangular base ORS and height OT. The position vectors of R and S are $-4\mathbf{i}+\mathbf{j}+3\mathbf{k}$ and $5\mathbf{i}+\mathbf{j}$ respectively. Find the possible coordinates of T if the volume of the pyramid $ORST$ is 35 units^3. [5]

[Volume of pyramid $=\frac{1}{3}\times$ base area $\times$ height]

Answers: (a)(i) $k|\underset{\sim}{b}|^2$; where k is a constant (ii) $AP=\frac{|\underset{\sim}{a}\times\underset{\sim}{b}|}{|\underset{\sim}{b}|}$, $BQ=\frac{|\underset{\sim}{a}\times\underset{\sim}{b}|}{|\underset{\sim}{a}|}$ (iii) $(-2,10,-6)$ and $(2,-10,6)$

2019 ASRJC P1 Q9



The diagram above shows an object with O at the centre of its rectangular base $ABCD$ where $AB=8$ cm and $BC=4$ cm. The top side of the object, $EFGH$ is a square with side 2 cm long and is parallel to the base. The centre of the top side is vertically above O at a height h cm.

(i) Show that the equation of the line BG may be expressed as $r=\begin{pmatrix}4\\-2\\0\end{pmatrix}+t\begin{pmatrix}-3\\1\\h\end{pmatrix}$, where t is a parameter. [1]

(ii) Find the sine of the angle between the line BG and the rectangular base $ABCD$ in terms of h. [2]

It is given that $h=6$.

(iii) Find the cartesian equation of the plane $BCFG$. [3]

(iv) Find the shortest distance from the point A to the plane $BCFG$. [2]

(v) The line l, which passes through the point A, is parallel to the normal of plane $BCFG$. Given that, the line l intersects the plane $BCFG$ at a point M, use your answer in part **(iv)** to find the shortest distance from point M to the rectangular base $ABCD$. [2]

Answers: (ii) $\frac{h}{\sqrt{10+h^2}}$ (iii) $2x+z=8$ (iv) $\frac{16}{\sqrt{5}}$ units (v) $\frac{16}{5}$