SIGMA NOTATION

2017 EJC PROMO Q7 (B) [MODIFIED]

Given that $\sum_{r=2}^{n}\frac{1}{r^2-1}=\frac{3}{4}-\frac{1}{2n}-\frac{1}{2(n+1)}$,

(ii) state $\sum_{r=2}^{\infty}\frac{1}{r^2-1}$,

(iii) find $\sum_{r=2}^{n-1}\frac{1}{r(r+2)}$.

Answers: (ii) $\frac{3}{4}$ (iii) $\frac{5}{12}-\frac{1}{2n}-\frac{1}{2(n+1)}$

2018 YJC PROMO Q13

The terms of geometric progression $u_1, u_2, u_3, u_4, \ldots$ are such that the sum to infinity is 81 and the sum of the first 4 terms is 80.

If $u_1 > 100$ and $n \geq 3$,

(i) Show that $\frac{3}{r}-\frac{6}{r+1}+\frac{3}{r+2}=\frac{6}{r(r+1)(r+2)}$. [1]

(ii) Hence show that $\sum_{r=1}^{N}\frac{1}{r(r+1)(r+2)}=\frac{1}{4}-\frac{1}{2(N+1)}+\frac{1}{2(N+2)}$. [3]

(iii) Give a reason why the series $\sum_{r=1}^{\infty}\frac{1}{r(r+1)(r+2)}$ converges and write down its value. [2]

(iv) Use your answer to **(ii)** to find $\sum_{r=3}^{N}\frac{1}{r(r-1)(r-2)}$ in terms of N. [2]

Answers: (iii) $\frac{1}{4}$ (iv) $\frac{1}{4}-\frac{1}{2(N-1)}+\frac{1}{2N}$

2019 DHS PROMO Q2

Using the result $\sum_{r=1}^{n} \frac{r}{2^r} = 2 - \frac{n+2}{2^n}$, show that $\sum_{r=1}^{n} (r-n)(2^{-r}+1)$ can be expressed in the form $C\left(1-\frac{1}{2^n}\right) + Dn(n+1)$, where C and D are constants to be determined. [4]

Answer: $C = 2$, $D = -\frac{1}{2}$

2017 VJC P1 Q8

It is given that $\sum_{r=1}^{n} \frac{r^2}{3^r} = \frac{3}{2} - \frac{n^2+3n+3}{2(3^n)}$.

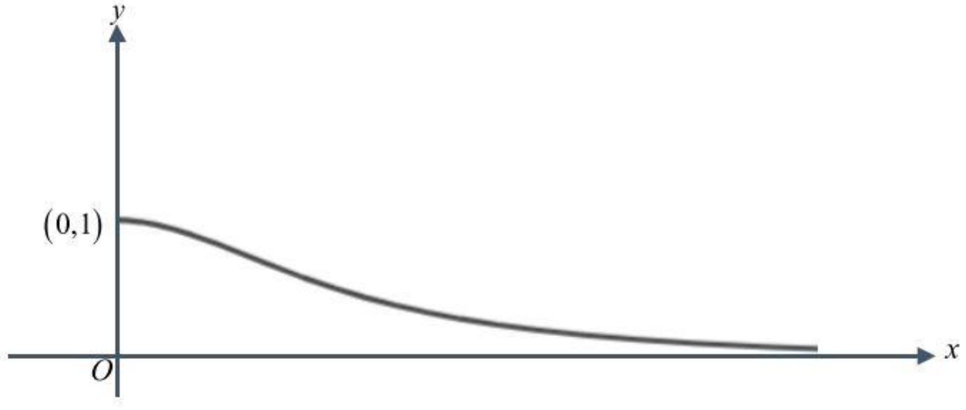
(i) Find $\sum_{r=1}^{\infty} \frac{r^2+(-1)^r}{3^r}$. [3]

(ii) Show that $\sum_{r=4}^{n} \frac{(r-2)^2}{3^{r-2}} = \frac{p}{q} - \frac{an^2-an+a}{2(3^{n-2})}$, where a, p and q are integers to be determined. [5]

Answers: (i) $\frac{5}{4}$ (ii) $p = 7, q = 6, a = 1$

2020 EJC P1 Q10

The diagram shows the graph of $y=\dfrac{1}{x^2+1}$ when $x>0$.



(i) Evaluate $\displaystyle\int_k^{k+1}\frac{1}{x^2+1}\,\mathrm{d}x$ for $k>0$, leaving your answer in terms of k. [2]

(ii) By considering appropriate rectangles on the interval $[k,k+1]$ for the curve $y=\dfrac{1}{x^2+1}$, show that

$$\frac{1}{(k+1)^2+1}<\tan^{-1}(k+1)-\tan^{-1}k<\frac{1}{k^2+1}\quad\text{for } k\in\mathbb{Z}^+.$$

[2]

(iii) Use the identity $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$ to show that

$$\tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy},\ \text{where } x>y>0.$$

[2]

(iv) By considering parts **(ii)** and **(iii)**, prove by the method of differences that

$$\sum_{k=1}^{n}\frac{1}{(k+1)^2+1}<\tan^{-1}\left(\frac{n}{n+2}\right)<\sum_{k=1}^{n}\frac{1}{k^2+1}$$

[4]

Answer: (i) $\tan^{-1}(k+1)-\tan^{-1}k$